

**Метод минимальной стоимости**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | ai |
| A1 | 20 2 | 3 | 10 2 | 4 | 30 |
| A2 | 3 | 30 2 | 5 | 10 1 | 40 |
| A3 | 4 | 3 | 20 2 | 6 | 20 |
| bj | 20 | 30 | 30 | 10 | 90 |

Z=2\*20+2\*10+2\*30+1\*10+2\*20=170

3+4-1=6>5 ­­- план вырожденный

**Метод аппроксимации Фогеля**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | ai | Δcij |
| A1 | 20 2 | 3 | 10 2 | 4 | 30 | 1, 1, 1 B |
| A2 | 3 | 30 2 | 5 | 10 1 | 40 | 1, 1, 4 B |
| A3 | 4 | 3 | 20 2 | 6 | 20 | 1, 1, 4 B |
| bj | 20 | 30 | 30 | 10 | 90 |  |
| Δcij | 1 B | 1 B | 3, 3 B | 3, 5 |  |  |

Z=2\*20+2\*10+2\*30+1\*10+2\*20=170

3+4-1=6>5 ­­- план вырожденный

**Проверка на оптимальность**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | ai |
| A1 | 20 2 | 3 | 10 2 | 4 | 30 |
| A2 | 3 | 30 2 | 5 | 10 1 | 40 |
| A3 | 4 | 0 3 | 20 2 | 6 | 20 |
| bj | 20 | 30 | 30 | 10 | 90 |

Z=2\*20+2\*10+2\*30+1\*10+2\*20=170

Составим систему уравнений потенциалов:

u1 + v1 = 2

u1 + v3 = 2

u2 + v2 = 2

u2 + v4 = 1

u3 + v2 = 3

u3 + v3 = 2

Полагая u1 = 0, найдем:

v1 = 2

v2 = 3 u2 = -1

v3 = 2 u3 = 0

v4 = 2

Проверяем на соответствие теореме 5:

u1 + v2 = 2 < 3

u1 + v4 = 2 < 4

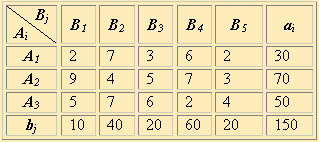
u2 + v1 = 1 < 3

u2 + v3 = 1 < 5

u3 + v1 = 2 < 4

u3 + v4 = 2 < 6

План оптимален; это единственный оптимальный план



**Метод минимальной стоимости**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai |
| A1 | 10 2 | 7 | 3 | 6 | 20 2 | 30 |
| A2 | 9 | 40 4 | 20 5 | 10 7 | 3 | 70 |
| A3 | 5 | 7 | 6 | 50 2 | 4 | 50 |
| bj | 10 | 40 | 20 | 60 | 20 | 150 |

Z=2\*10 + 2\*20 + 4\*40 +5\*20 + 7\*10 + 2\*50 = 490

**Метод аппроксимации Фогеля**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai | Δcij |
| A1 | 10 2 | 7 | 10 3 | 10 6 | 2 | 30 | 0, 1, 1, 1, - B |
| A2 | 9 | 40 4 | 10 5 | 7 | 20 3 | 70 | 1, 1, 2, 2, - B |
| A3 | 5 | 7 | 6 | 50 2 | 4 | 50 | 2 B |
| bj | 10 | 40 | 20 | 60 | 20 | 150 |  |
| Δcij | 3, 7 B | 3, 3 B | 2, 2 B | 4, 3 B | 1, 1 B |  |  |

Z = 2\*10 + 4\*40 + 3\*10 + 5\*10 + 6\*10 + 2\*50 + 3\*20 = 480

Проверка на оптимальность:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai |
| A1 | 10 2 | 7 | 10 3 | 10 6 | 2 | 30 |
| A2 | 9 | 40 4 | 10 5 | 7 | 20 3 | 70 |
| A3 | 5 | 7 | 6 | 50 2 | 4 | 50 |
| bj | 10 | 40 | 20 | 60 | 20 | 150 |

Составим систему уравнений потенциалов:

u1 + v1 =2

u1 + v3 =3

u1 + v4 =6

u2 + v2 =4

u2 + v3 =5

u2 + v5 =3

u3 + v4 =2

Полагая u1 = 0, найдем:

v1 = 2

v2 = 2 u2 = 2

v3 = 3 u3 = -4

v4 = 6

v5 = 1

Проверяем на соответствие теореме 5:

u1 + v2 = 2 < 7

u1 + v5 =1 < 2

u2 + v1 =4 < 9

u2 + v4 = 8 > 7

u3 + v1 =-2 < 5

u3 + v2 =-2 < 7

u3 + v3 =-1 < 6

u3 + v5 =-3 < 4

План не оптимален

Сдвиг по циклу, вершины в точках A2B4 (+), A2B3 (-), A1B3 (+), A1B4 (+). Сдвиг на 10

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai |
| A1 | 10 2 | 7 | 20 3 | 6 | 2 | 30 |
| A2 | 9 | 40 4 | 5 | 10 7 | 20 3 | 70 |
| A3 | 5 | 7 | 6 | 50 2 | 4 | 50 |
| bj | 10 | 40 | 20 | 60 | 20 | 150 |

u2 + v4 = 7

Полагая u1 = 0, найдем:

v4 = 5

u3 = -3

Проверяем на соответствие теореме 5:

u1 + v2 = 2 < 7

u1 + v5 =1 < 2

u2 + v1 =4 < 9

u2 + v4 = 7 = 7

u3 + v1 =-1 < 5

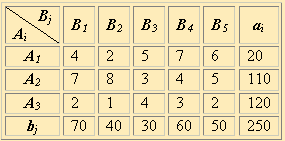
u3 + v2 =-1 < 7

u3 + v3 =0 < 6

u3 + v5 =-2 < 4

Z = 2\*10 + 4\*40 + 3\*20 + 2\*50 + 3\*20 + 7\*10 = 470

План оптимален; существуют другие оптимальные планы.



**Метод минимальной стоимости**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai |
| A1 | 4 | 20 2 | 5 | 7 | 6 | 20 |
| A2 | 7 | 8 | 30 3 | 60 4 | 20 5 | 110 |
| A3 | 70 2 | 20 1 | 4 | 3 | 30 2 | 120 |
| bj | 70 | 40 | 30 | 60 | 50 | 250 |

Z=2\*20+3\*30+4\*60+5\*20+2\*70+1\*20+2\*30=690

**Метод двойного предпочтения**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai |
| A1 | 4 | 2 V | 5 | 7 | 20 6 | 20 |
| A2 | 7 | 8 | 30 3 VV | 50 4 | 30 5 | 110 |
| A3 | 70 2 V | 40 1 VV | 4 | 10 3 V | 2 V | 120 |
| bj | 70 | 40 | 30 | 60 | 50 | 250 |

Z = 2\*70 + 1\*40 + 3\*30 + 4\*50 +3\*10 + 6\*20 + 5\*30 = 770

**Метод аппроксимации Фогеля**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai | Δcij |
| A1 | 4 | 20 2 | 5 | 7 | 6 | 20 | 2, 2 B |
| A2 | 20 7 | 8 | 30 3 | 60 4 | 5 | 110 | 1, 2, 2, 4, - B |
| A3 | 50 2 | 20 1 | 4 | 3 | 50 2 | 120 | 1, 1, 1 B |
| bj | 70 | 40 | 30 | 60 | 50 | 250 |  |
| Δcij | 2, 5, 3 B | 1, 7 B | 1, 1, 2 B | 1, 1, 3 B | 3 B |  |  |

Z = 7\*20 + 2\*50 + 2\*20 + 1\*20 + 3\*30 + 4\*60 + 2\*50 = 730

Проверка на оптимальность:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai |
| A1 | 4 | 20 2 | 5 | 7 | 6 | 20 |
| A2 | 7 | 8 | 30 3 | 60 4 | 20 5 | 110 |
| A3 | 70 2 | 20 1 | 4 | 3 | 30 2 | 120 |
| bj | 70 | 40 | 30 | 60 | 50 | 250 |

Z=2\*20+3\*30+4\*60+5\*20+2\*70+1\*20+2\*30=690

u1+v2=2

u2+v3=3

u2+v4=4

u2+v5=5

u3+v1=2

u3+v2=1

u3+v5=2

Полагая u1 = 0, найдем:

v1 = 3

v2 = 2 u2 = 2

v3 = 1 u3 = -1

v4 = 2

v5 = 3

Проверяем на соответствие теореме 5:

u1 + v1 = 3 < 4

u1 + v3 = 1 < 5

u1 + v4 = 2 < 7

u1 + v5 = 3 < 6

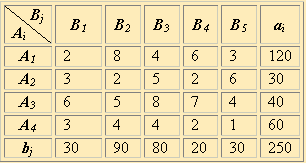
u2 + v1 = 5 < 7

u2 + v2 = 4 < 8

u3 + v3 = 0 < 4

u3 + v4 = 1 < 3

План оптимален; это единственный оптимальный план



**Метод минимальной стоимости**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai |
| A1 | 30 2 | 20 8 | 70 4 | 6 | 3 | 120 |
| A2 | 3 | 30 2 | 5 | 2 | 6 | 30 |
| A3 | 6 | 40 5 | 8 | 7 | 4 | 40 |
| A4 | 3 | 4 | 10 4 | 20 2 | 30 1 | 60 |
| bj | 30 | 90 | 80 | 20 | 30 | 250 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai |
| A1 | 30 2 | 10 8 | 80 4 | 6 | 3 | 120 |
| A2 | 3 | 30 2 | 5 | 2 | 6 | 30 |
| A3 | 6 | 40 5 | 8 | 7 | 4 | 40 |
| A4 | 3 | 10 4 | 4 | 20 2 | 30 1 | 60 |
| bj | 30 | 90 | 80 | 20 | 30 | 250 |

Z=2\*30+8\*10+4\*80+2\*30+5\*40+4\*10+2\*20+1\*30=830

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai |
| A1 | 30 2 | 8 | 80 4 | 6 | 10 3 | 120 |
| A2 | 3 | 30 2 | 5 | 2 | 6 | 30 |
| A3 | 6 | 40 5 | 8 | 7 | 4 | 40 |
| A4 | 3 | 20 4 | 4 | 20 2 | 20 1 | 60 |
| bj | 30 | 90 | 80 | 20 | 30 | 250 |

Z=2\*30+4\*80+3\*10+2\*30+5\*40+4\*20+2\*20+1\*20=810

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai |
| A1 | 30 2 | 8 | 80 4 | 6 | 3 | 120 |
| A2 | 3 | 30 2 | 5 | 10 2 | 6 | 30 |
| A3 | 6 | 40 5 | 8 | 7 | 4 | 40 |
| A4 | 3 | 20 4 | 4 | 10 2 | 30 1 | 60 |
| bj | 30 | 90 | 80 | 20 | 30 | 250 |

**Z=2\*30+4\*80+2\*30+2\*10+5\*40+4\*20+2\*10+1\*30=790**

Z=2\*30+8\*20+4\*70+2\*30+5\*40+4\*10+2\*20+1\*30=870

**Метод аппроксимации Фогеля**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai | Δcij |
| A1 | 30 2 | 10 8 | 80 4 | 6 | 3 | 120 | 1, 1, 2, 1 |
| A2 | 3 | 30 2 | 5 | 2 | 6 | 30 | 0, 1, 1, 3 B |
| A3 | 6 | 40 5 | 8 | 7 | 4 | 40 | 1, 1, 1, 3 B |
| A4 | 3 | 10 4 | 4 | 20 2 | 30 1 | 60 | 1, 2, 1, 0 B |
| bj | 30 | 90 | 80 | 20 | 30 | 250 |  |
| Δcij | 1 B | 2, 1, - | 0, 0, - | 4 B | 2 B |  |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai | Δcij |
| A1 | 30 2 | 10 8 | 80 4 | 6 | 3 | 120 | 1, 1, 2, 4, - |
| A2 | 3 | 30 2 | 5 | 2 | 6 | 30 | 0 B |
| A3 | 6 | 40 5 | 8 | 7 | 4 | 40 | 1, 1, 1, 3, - |
| A4 | 3 | 10 4 | 4 | 20 2 | 30 1 | 60 | 1, 2, 1, 0, - |
| bj | 30 | 90 | 80 | 20 | 30 | 250 |  |
| Δcij | 1, 1 B | 2, 1, 1 | 0, 0 B | 0, 4 B | 2, 2 B |  |  |

Z=2\*30+8\*10+4\*80+2\*30+5\*40+4\*10+2\*20+1\*30= 830

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai |
| A1 | 30 2vv | 20 8 | 70 4v | 6 | 3 | 120 |
| A2 | 3 | 30 2vv | 5 | 2vv | 6 | 30 |
| A3 | 6 | 40 5 | 8 | 7 | 4v | 40 |
| A4 | 3 | 4 | 10 4v | 20 2v | 30 1vv | 60 |
| bj | 30 | 90 | 80 | 20 | 30 | 250 |

Z=2\*30+8\*20+4\*70+2\*30+5\*40+4\*10+2\*20+1\*30=870

Проверка на оптимальность:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai |
| A1 | 30 2 | 8 | 80 4 | 6 | 3 | 120 |
| A2 | 3 | 30 2 | 5 | 10 2 | 6 | 30 |
| A3 | 6 | 40 5 | 8 | 7 | 4 | 40 |
| A4 | 3 | 20 4 | 4 | 10 2 | 30 1 | 60 |
| bj | 30 | 90 | 80 | 20 | 30 | 250 |

**Z=2\*30+4\*80+2\*30+2\*10+5\*40+4\*20+2\*10+1\*30=790**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai |
| A1 | 30 2 | 10 8 | 80 4 | 6 | 3 | 120 |
| A2 | 3 | 30 2 | 5 | 2 | 6 | 30 |
| A3 | 6 | 40 5 | 8 | 7 | 4 | 40 |
| A4 | 3 | 10 4 | 4 | 20 2 | 30 1 | 60 |
| bj | 30 | 90 | 80 | 20 | 30 | 250 |

Z=2\*30+8\*10+4\*80+2\*30+5\*40+4\*10+2\*20+1\*30=830

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai |
| A1 | 30 2vv | 20 8 | 70 4v | 6 | 3 | 120 |
| A2 | 3 | 30 2vv | 5 | 2vv | 6 | 30 |
| A3 | 6 | 40 5 | 8 | 7 | 4v | 40 |
| A4 | 3 | 4 | 10 4v | 20 2v | 30 1vv | 60 |
| bj | 30 | 90 | 80 | 20 | 30 | 250 |

Z=2\*30+8\*20+4\*70+2\*30+5\*40+4\*10+2\*20+1\*30=870

u1+v1=2

u1+v2=8

u1+v3=4

u2+v2=2

u3+v2=5

u4+v3=4

u4+v4=2

u4+v5=1

Полагая u1 = 0, найдем:

v1 = 2

v2 = 8 u2 = -6

v3 = 4 u3 = -3

v4 = 2 u4 =0

v5 = 1

Проверяем на соответствие теореме 5:

u1+v4=2 < 6

u1+v5=1 > 3

u2+v1=-4 < 3

u2+v3=-2 < 5

u2+v4=-4 < 2

u2+v5=-5 < 6

u3+v1=-1 < 6

u3+v3=1 < 8

u3+v4=-1 < 7

u3+v5=-2 < 4

u4+v1=2 < 3

u4+v2=8 > 4

План не оптимален

Сдвиг по циклу, вершины в точках A2B4 (+), A2B3 (-), A1B3 (+), A1B4 (+). Сдвиг на 10

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai |
| A1 | 30 2 | 10 8 | 80 4 | 6 | 3 | 120 |
| A2 | 3 | 30 2 | 5 | 2 | 6 | 30 |
| A3 | 6 | 40 5 | 8 | 7 | 4 | 40 |
| A4 | 3 | 10 4 | 4 | 20 2 | 30 1 | 60 |
| bj | 30 | 90 | 80 | 20 | 30 | 250 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ai \ Bj | B1 | B2 | B3 | B4 | B5 | ai |
| A1 | 30 2 | 20 8 | 70 4 | 6 | 3 | 120 |
| A2 | 3 | 30 2 | 5 | 2 | 6 | 30 |
| A3 | 6 | 30 5 | 10 8 | 7 | 4 | 40 |
| A4 | 3 | 10 4 | 4 | 20 2 | 30 1 | 60 |
| bj | 30 | 90 | 80 | 20 | 30 | 250 |

u1+v1=2

u1+v2=8

u1+v3=4

u2+v2=2

u3+v2=5

u3+v3=8

u4+v2=4

u4+v4=2

u4+v5=1

Полагая u1 = 0, найдем:

v1 =2

v2 =8 u2 = -6

v3 = 4 u3 = 4

v4 =6 u4 =-4

v5 =5

Проверяем на соответствие теореме 5:

u1+v4=6 < 6

u1+v5=5 > 3

u2+v1=-4 < 3

u2+v3=-2 < 5

u2+v4=-4 < 2

u2+v5=-5 < 6

u3+v1=-1 < 6

u3+v4=-1 < 7

u3+v5=-2 < 4

u4+v1=2 < 3

u4+v3=8 > 4